H. Algebra 2

Unit 3 Review

Name: Key Date: Hour:

Module 5 – Polynomials Functions

5.1 Graphing Cubic Functions

Describe, in words, the transformations applied to the graph of $f(x) = x^3$ to produce the graph of $g(x)$.

1. $g(x) = -3(x+1)^3 - 9$

Left 1 Vert St. factor 3 reflected over X-axis down 9

2. $g(x) = \frac{1}{4}(-x)^3 + 5$

reflected over y-axis Vert Comp 1/4 up 5

3. $g(x) = (2(x-1))^3$

Right 1 Horz Comp 1/2

4. $g(x) = \left(\frac{1}{3}(x+4)\right)^3 - 7$

Horz Stretch 3 left 4 down 7

4. $g(x) = a(x-h)^3 + k$

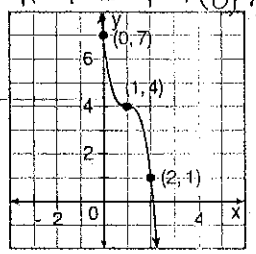
$7 = a(0-1)^3 + 4$

$3 = a(-1)$

$a = -3$

$g(x) = -3(x-1)^3 + 4$

h=1 k=4 P(0,7)



5. $g(x) = \left(\frac{1}{b}(x-h)\right)^3 + k$

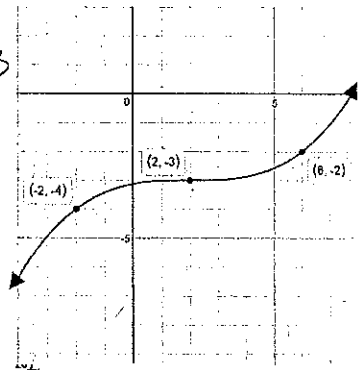
h=2 k=-3 P(6,-2)

$-2 = \left(\frac{1}{b}(6-2)\right)^3 - 3$

$1 = (1/b(4))^3$

$1 = \frac{4}{b}$ $b = 4$

$g(x) = \left(\frac{1}{4}(x-6)\right)^3 - 3$



6. End Behavior.

As $x \to \infty$ $f(x) \to -\infty$ As $x \to -\infty$ $f(x) \to \infty$

7. Even or Odd degree? odd

8. Positive or negative leading coeff? negative

9. Zeros and multiplicities. $x = -4$ (mult 2), 0, 2, 5

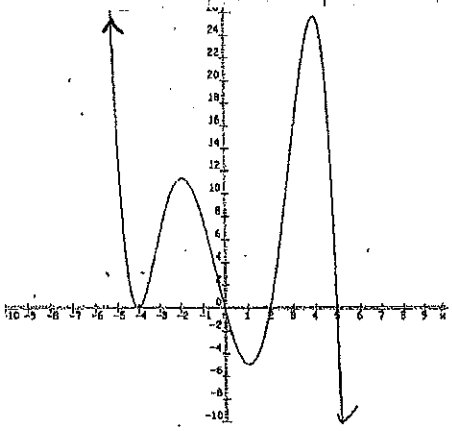
10. Number of turning Points 4

11. # of Global Max 0 (if arrows)

12. # of Local Max 2

13. # of Global Min 0 (if arrows)

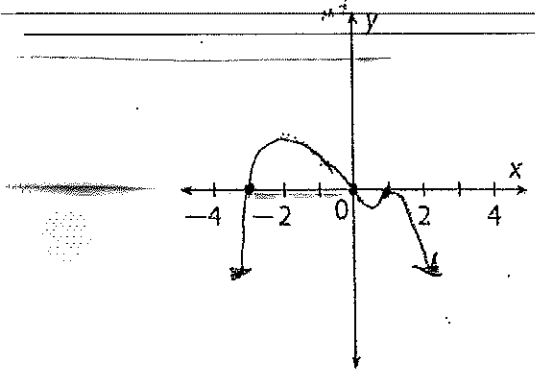
14. # of Local Min 2



Use the end behavior, x-intercepts, and y-intercept to sketch the graph of the function.

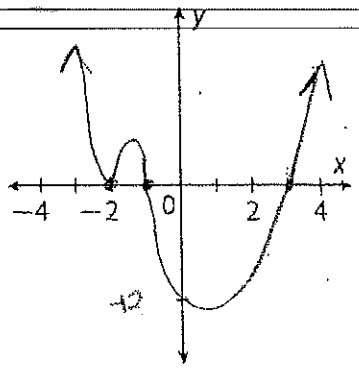
15. $g(x) = -x(x-1)^2(x+3)$

↓ $x = 0, 1, -3$ $y = 0$



16. $g(x) = (x-3)(x+1)(x+2)^2$

↑ $x = 3, -1, -2$ $y = -12$



Module 6 – Polynomials

6.1 Adding and Subtracting Polynomials

Add or Subtract. Write your answers in standard form.

1. $(x - 4 + 6x^2 + 8x^3) + (17x - 9x^2 + 6)$

$8x^3 - 3x^2 + 18x + 2$

2. $(5x^3 + 7x^2 - 10x^4 + x^{12}) - (3x^{12} + 4x^2 + 1)$

$5x^3 + 7x^2 - 10x^4 + x^{12} - 3x^{12} - 4x^2 - 1$

$-2x^{12} - 10x^4 + 5x^3 + 3x^2 - 1$

3. $(8x^5 + x^3 - x) - (x + 7x^5)$

$x^5 + x^3 - 2x$

4. $(12x + 11x^2 - 10x^3 + 4) + (-9x^3 - 14x + 5)$

$-19x^3 + 11x^2 - 2x + 9$

5. A rectangular field has a perimeter of $(3x^3 - 12x^2 + 10x - 75)$ miles and a length of x miles.

a) Write an equation for the width of the field in terms of the length.

$2x + 2w = 3x^3 - 12x^2 + 10x - 75$

$2w = 3x^3 - 12x^2 + 8x - 75$

$w = \frac{3}{2}x^3 - 6x^2 + 4x - \frac{75}{2}$

b) Find the width of the field when the length is 5 miles.

$w(5) = \frac{3}{2}(5)^3 - 6(5)^2 + 4(5) - \frac{75}{2}$

$w(5) = \frac{375}{2} - 150 + 20 - \frac{75}{2}$

$w(5) = \frac{300}{2} - 150 + 20$

$w(5) = 150 - 150 + 20$

$w(5) = 20$

6.2 Multiplying Polynomials

Multiply. Write your answers in standard form.

1. $(x^2 - 2x + 6)(x^3 - 5x)$

$x^5 - 5x^3 - 2x^4 + 10x^2 + 6x^3 - 30x$

$x^5 - 2x^4 + x^3 + 10x^2 - 30x$

2. $(x^4 + 2x^2)(4x - 3)$

$4x^5 - 3x^4 + 8x^3 - 6x^2$

3. $(2xy - 4x)(y^2 - 3xy + y)$

$2xy^3 - 6x^2y^2 + 2xy^2 - 4xy^2 + 12x^2y - 4xy$

$2xy^3 - 6x^2y^2 - 2xy^2 + 12x^2y - 4xy$

4. $(3p^2 + 4p + 5)(2 - p)$

$6p^2 - 3p^3 + 8p - 4p^2 + 10 - 5p$

$-3p^3 + 2p^2 + 3p + 10$

6.3 The Binomial Theorem

Expand using the Binomial Theorem

1. $(x - 2)^5$

$1x^5 + 5x^4(-2) + 10x^3(-2)^2 + 10x^2(-2)^3 + 5x(-2)^4 + (-2)^5$

$x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$

2. $(3m - n)^4$

$1(3m)^4 + 4(3m)^3n + 6(3m)^2(n)^2 + 4(3m)(n)^3 + (-n)^4$

$81m^4 - 108n + 54m^2n^2 - 3mn^3 + n^4$

3. $(x + 2y)^6$

$1x^6 + 6x^5(2y) + 15x^4(2y)^2 + 20x^3(2y)^3 + 15x^2(2y)^4 + 6x(2y)^5 + (2y)^6$

$x^5 + 12x^5y + 60x^4y^2 + 160x^3y^3 + 240x^2y^4 + 192xy^5 + 64y^4$

Find the specific term in each binomial expansion.

4. $(x - 2)^5$; 4th term

$10(x)^2(-2)^3$

$-80x^2$

5. $(3x - 1)^4$; 3rd term

$6(3x)^2(-1)^2$

$54x^2$

6. $(3 + x)^7$; 5th term

$35(3)^3(x)^4$

$945x^4$

6.4 Factoring Polynomials

Factor completely.

1. $10x^3 - 4x^2 - 15x + 6$

$2x^2(5x-2) - 3(5x-2)$

$(2x^2-3)(5x-2)$

2. $6x^3 + 3x^2 - 4x - 2$

$3x^2(2x+1) - 2(2x+1)$

$(3x^2-2)(2x+1)$

3. $125x^3 - 64$ $a=5x$ $b=4$

$(5x-4)(25x^2+20x+16)$

4. $10x^4 + 9x^2 - 40$ $x -400 +9$ $-16 \cdot 25$

$(10x^4-16x^2)+(25x^2-40)$

$2x^2(5x^2-8)+5(5x^2-8)$

$(2x^2+5)(5x^2-8)$

5. $15x^3 + 20x^2 - 3x - 4$

$5x^2(3x+4) - 1(3x+4)$

$(5x^2-1)(3x+4)$

6. $100x^4 - 25$

$25(4x^4-1)$

$25(2x^2-1)(2x^2+1)$

7. $2x^4 + 7x^3 + 5x^2$ $\times$ $+$ 10 7

$x^2(2x^2+7x+5)$

$x^2((2x^2+2x)+(5x+5))$

$x(2x(x+1)+5(x+1))$

$x(x+1)(2x+5)$

8. $x^3 + x^2 - 12x$

$x(x^2+x-12)$

$x(x+4)(x-3)$

9. $128x^3 + 54$

$2(64x^3+27)$ $a=4x$ $b=3$

$2(4x+3)(16x^2-12x+9)$

6.5 Dividing Polynomials (10) See End!

Use long division to divide the polynomials. Write your answers as *dividend* = (*divisor*)(*quotient*) + *remainder*.

1. $(5x^4 - 42x^3 - 77x^2 - 5x + 10) \div (5x + 3)$

$(5x^4-42x^3-77x^2-5x+10) = (5x+3)(x^3-9x^2-10x+5) - 5$

2. $(4x^3 + 2x^2 + 3x + 5) \div (x^2 + 3x + 1)$

$(4x^3+2x^2+3x+5) = (x^2+3x+1)(4x-10) + 29x + 15$

3. $(9x^4 - 9x^3 + 3) \div (9x - 9)$

$(9x^4-9x^3+3) = (9x-9)(x^3)+3$

4. $(2x^4 + x^3 - 31x^2 - 27x + 25) \div (2x + 7)$

$(2x^4+x^3-31x^2-27x+25) = (2x+7)(x^3-3x^2-5x+4) - 3$

Use synthetic division to divide the polynomials. Write your answers as *dividend* = (*divisor*)(*quotient*) + *remainder*.

5. $(x^4 - 2x^3 + 5x - 15) \div (x - 2)$

$2 \mid 1 \;\; -2 \;\; 0 \;\; 5 \;\; -15$

$\quad\quad\;\; 2 \;\; 0 \;\; 0 \;\; 10$

$\quad 1 \;\; 0 \;\; 0 \;\; 5 \;\; -5$

$(x^4-2x^3+5x-15) = (x-2)(x^3+5) - 5$

6. $(x^4 + 15x^3 + 61x^2 + 43x + 25) \div (x + 8)$

$-8 \mid 1 \;\; 15 \;\; 61 \;\; 43 \;\; 25$

$\quad\quad\;\; -8 \;\; -56 \;\; -40 \;\; -24$

$\quad 1 \;\; 7 \;\; 5 \;\; 3 \;\; 1$

$(x^4+15x^3+61x^2+43x+25) = (x+8)(x^3+7x^2+5x+3) + 1$

Use synthetic substitution to evaluate $p(x)$ for the given value.

7. $p(x) = -2x^4 + x^3 - 15x - 10; x = 2$

$2 \mid -2 \;\; 1 \;\; 0 \;\; -15 \;\; -10$

$\quad\quad\;\; -4 \;\; -6 \;\; -12 \;\; -54$

$\quad -2 \;\; -3 \;\; -6 \;\; -27 \mid -64$

$p(2) = -64$

8. $p(x) = -x^5 - 4x^2 + 1; x = -4$

$-4 \mid -1 \;\; 0 \;\; 0 \;\; -4 \;\; 0 \;\; 1$

$\quad\quad\;\; 4 \;\; -16 \;\; 64 \;\; -256$

$\quad -1 \;\; 4 \;\; -16 \;\; 64 \;\; -256 \mid -1024$

$p(-4) = -1024$

Determine whether the given binomial is a factor of $p(x)$. If it is, completely factor $p(x)$.

9. $(x - 3); p(x) = x^3 + 3x^2 - 34x + 48$

$3 \mid 1 \;\; 3 \;\; -34 \;\; 48$

$\quad\quad\;\; 3 \;\; 18 \;\; -48$

$\quad 1 \;\; 6 \;\; -16 \;\; 0$

$(x-3)(x^2+6x-16)$ $\quad -16 \cdot -6$, $8 \cdot 2$

$(x-3)(x+8)(x-2)$

10. $(x + 5); p(x) = -2x^3 + 6x^2 - x$

$-5 \mid -2 \;\; 6 \;\; -1 \;\; 0$

$\quad\quad\;\; 10 \;\; -80 \;\; 405$

$\quad -2 \;\; 16 \;\; -81 \mid 405$

NO!

Module 7 – Polynomial Equations

7.1 Finding Rational Solutions of Polynomial Equations

Solve each polynomial equation by factoring.

1. $4x^3 + x^2 - 4x - 1 = 0$

$x^2(4x+1) - 1(4x+1) = 0$ $\boxed{x = -1, -\frac{1}{4}, 1}$

$(x^2-1)(4x+1) = 0$

$x^2 - 1 = 0 \quad x = \pm 1 \quad x = -\frac{1}{4}$

2. $x^5 - 2x^4 - 24x^3 = 0$

$x^3(x^2 - 2x - 24) = 0$

$x^3(x-6)(x+4) = 0$

$\boxed{x = 0, -4, 6}$

3. $3x^5 + 18x^4 - 21x^3 = 0$

$3x^3(x^2 + 6x - 7) = 0$

$3x^3(x+7)(x-1) = 0$

$\boxed{x = 0, -7, 1}$

4. $-x^4 + 2x^3 + 8x^2 = 0$

$-x^2(x^2 - 2x - 8) = 0$

$-x^2(x-4)(x+2) = 0$

$\boxed{x = 0, 4, -2}$

List all possible rational zeros of the function, then write the function in factored form.

5. $f(x) = x^3 + 3x^2 + 3x + 1$ $\frac{m}{n} = \pm 1$

$\frac{m}{n}$	1	3	3	1
1	1	4	7	—
−1	1	2	1	0

$f(x) = (x+1)(x^2 + 2x + 1)$

$f(x) = (x+1)(x+1)^2$

$\boxed{f(x) = (x+1)^3}$

6. $f(x) = x^3 + 5x^2 - 8x - 48$ $\frac{m}{n} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 16, \pm 24, \pm 48$

$\frac{m}{n}$	1	5	−8	−48
1	1	6	−2	—
−1	1	4	−12	—
2	1	7	−6	—
−2	1	3	−14	—
3	1	8	16	0

$f(x) = (x-3)(x^2 + 8x + 16)$

$\boxed{f(x) = (x-3)(x+4)^2}$

List all possible rational roots of each equation, then find the actual roots.

7. $x^3 + 10x^2 + 17x = 28$

$x^3 + 10x^2 + 17x - 28 = 0$

$\boxed{\frac{m}{n} = \pm 1, \pm 2, \pm 4, \pm 7, \pm 14, \pm 28}$

$\frac{m}{n}$	1	10	17	−28
1	1	11	28	0

$f(x) = (x-1)(x^2 + 11x + 28)$

$f(x) = (x-1)(x+4)(x+7)$

$0 = (x-1)(x+4)(x+7)$

$\boxed{x = -7, -4, 1}$

8. $3x^3 + 10x^2 - 27x = 10$

$3x^3 + 10x^2 - 27x - 10 = 0$

$m = \pm 1, \pm 2, \pm 5, \pm 10$

$n = \pm 1, \pm 3$

$\frac{m}{n} = \pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 5, \pm \frac{5}{3}, \pm 10, \pm \frac{10}{3}$

$\frac{m}{n}$	3	10	−27	−10
1	3	13	−1	—
−1	3	7	−34	—
2	3	16	5	0

$0 = (x-2)(3x^2 + 16x + 5)$

$0 = (x-2)(3x^2 + 1x + 15x + 5)$

$0 = (x-2)[x(3x+1) + 5(3x+1)]$

$0 = (x-2)(3x+1)(x+5)$

$\boxed{x = -\frac{1}{3}, -5, 2}$

9. An engineer is designing a storage compartment in a spacecraft. The compartment must be 2 meters longer than it is wide, and its depth must be 1 meter less than its width. The volume of the compartment must be 8 cubic meters.

a) Write an equation to model the volume of the compartment.

$V(x) = x(x-1)(x+2)$ $V(x) = x(x^2 + x - 2)$ $V(x) = x^3 + x^2 - 2x$

$8 = x^3 + x^2 - 2x$ $0 = x^3 + x^2 - 2x - 8$

$\boxed{V(x) = x^3 + x^2 - 2x - 8}$

$x-1 \overline{)\, x+2}$ (quotient x)

b) List all possible rational roots.

$\boxed{\pm 1 \quad \pm 2 \quad \pm 4}$

c) Use synthetic division to find the roots of the polynomial equation. Are the roots all rational numbers?

$\frac{m}{n}$	1	1	−2	−8
1	1	2	0	—
−1	1	0	−2	—
2	1	3	4	0

$V(x) = (x-2)(x^2 + 3x + 4)$

$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(4)}}{2}$

$x = \frac{-3 \pm \sqrt{9 - 16}}{2}$

$\boxed{x = \frac{-3 \pm \sqrt{-7}}{2}, 2}$ $\boxed{\text{No}}$

7.2 Finding Complex Solutions of Polynomial Equations

Write the simplest polynomial function with the given roots.

1. 1, 4, and -3

$f(x)=(x-1)(x-4)(x+3)$

$f(x)=(x^2-5x+4)(x+3)$

$f(x)=x^3+3x^2-5x^2-15x+4x+12$

$\boxed{f(x)=x^3-2x^2-11x+12}$

2. $\frac{1}{2}$, 5, and -2

$f(x)=(x-\frac{1}{2})(x-5)(x+2)$

$f(x)=(x-\frac{1}{2})(x^2-3x-10)$

$f(x)=(x^3-3x^2-10x-\frac{1}{2}x^2+\frac{3}{2}x+5)$

$\boxed{f(x)=x^3-3\frac{1}{2}x^2-8\frac{1}{2}x+5}$

3. $2i$, $\sqrt{3}$, and 4

$f(x)=(x-2i)(x+2i)(x-\sqrt{3})(x+\sqrt{3})(x-4)$

$f(x)=(x^2+4)(x^2-3)(x-4)$

$f(x)=(x^4+x^2-12)(x-4)$

$\boxed{f(x)=x^5-4x^4+x^3-4x^2-12x+48}$

4. $\sqrt{2}$, -5, and $-3i$

$f(x)=(x-\sqrt{2})(x+\sqrt{2})(x+5)(x-3i)(x+3i)$

$f(x)=(x^2-2)(x+5)(x^2+9)$

$f(x)=(x^3+5x^2-2x-10)(x^2+9)$

$f(x)=x^5+9x^3+5x^4+45x^2-2x^3-18x-10x^2-90$

$\boxed{f(x)=x^5+5x^4+7x^3+35x^2-18x-90}$

Solve each equation by finding all roots.

5. $x^4-2x^3-14x^2-2x-15=0$ $\frac{m}{n}=\pm1,\pm3,\pm5,\pm15$

$\frac{m}{n}$	1	−2	−14	−2	−15
1	1	−1	−15	−17	
−1	1	−3	−10	8	
3	1	1	−11	−35	
−3	1	−5	1	−5	0

$0=(x+3)(x^3-5x^2+x-5)$

$0=(x+3)(x^2(x-5)+1(x-5))$

$0=(x+3)(x^2+1)(x-5)$

$\boxed{x=-3,5,\pm i}$ $x^2=-1$ $x=\pm i$

6. $x^4-16=0$ $(x^2-4)(x^2+4)=0$

$(x-2)(x+2)(x^2+4)=0$

$\boxed{x=2,-2,2i,-2i}$ $x^2=-4$ $x=\pm2i$

7. $x^4+4x^3+4x^2+64x-192=0$ $\frac{m}{n}=\pm1,\pm2,\pm3,\pm4$

$\frac{m}{n}$	1	4	4	64	−192
1	1	5	9	73	
−1	1	3	1	63	
2	1	6	16	96	0

$0=(x-2)(x^3+6x^2+16x+96)$

$0=(x-2)(x^2(x+6)+16(x+6))$

$0=(x-2)(x+6)(x^2+16)$

$0=x-2$ $0=x+6$ $0=x^2+16$

$\boxed{x=2,-6,\pm4i}$ $x^2=-16$ $x=\pm4i$

8. $x^3-64=0$ $(x-4)(x^2+4x+16)=0$

$x=\frac{-4\pm\sqrt{4^2-4(1)(16)}}{2(1)}$

$x=\frac{-4\pm\sqrt{16-64}}{2(1)}$

$x=\frac{-4\pm\sqrt{48}}{2}$ $\boxed{x=-2\pm2i\sqrt{3}}$

$x=\frac{-4\pm4i\sqrt{3}}{2}$ $\boxed{x=4}$

9. An electrical circuit is designed such that its output voltage, V, measured in volts, can be either positive or negative. The voltage of the circuit passes through zero at $t = 1, 2,$ and 7 seconds. Write the simplest polynomial describing the voltage $V(t)$.

$V(t)=(t-1)(t-2)(t-7)$

$V(t)=(t^2-3t+2)(t-7)$

$V(t)=t^3-7t^2-3t^2+21t+2t-14$

$\boxed{V(t)=t^3-10t^2+23t-14}$

6.5

(1)

$$x^3 - 9x^2 - 10x + 5$$
$$5x+3 \;\big|\; 5x^4 - 42x^3 - 77x^2 - 5x + 10$$
$$-(5x^4 + 3x^3)$$
$$-45x^3 - 77x^2$$
$$-(-45x^3 - 27x^2)$$
$$-50x^2 - 5x$$
$$-(-50x^2 - 30x)$$
$$25x + 10$$
$$-(25x + 15)$$
$$-5$$

$$5x^4 - 42x^3 - 77x^2 - 5x + 10 = (5x+3)(x^3 - 9x^2 - 10x + 5) - 5$$

(2)

$$4x - 10$$
$$x^2 + 3x + 1 \;\big|\; 4x^3 + 2x^2 + 3x + 5$$
$$-(4x^3 + 12x^2 + 4x)$$
$$-10x^2 - x + 5$$
$$-(-10x^2 - 30x - 10)$$
$$29x + 15$$

$$(4x^3 + 2x^2 + 3x + 5) = (x^2 + 3x + 1)(4x - 10) + 29x + 15$$

(3)

$$x^3 + 0x^2 + 0x + 0$$
$$9x - 9 \;\big|\; 9x^4 - 9x^3 + 0x^2 + 0x + 3$$
$$-(9x^4 - 9x^3)$$
$$0x^3 + 0x^2 + 0x + 3$$
$$-(0x^3 + 0x^2 + 0x + 0)$$
$$3$$

$$(9x^4 - 9x^3 + 3) = (9x - 9)(x^3) + 3$$

6.5 (cont.)

(4)

$$x^3 - 3x^2 - 5x + 4$$

$2x+7 \;\overline{)\; 2x^4 + x^3 - 31x^2 - 27x + 25}$

$-(2x^4 + 7x^3)$

$-6x^3 - 31x^2$

$-(-6x^3 - 21x^2)$

$-10x^2 - 27x$

$-(-10x^2 - 35x)$

$8x + 25$

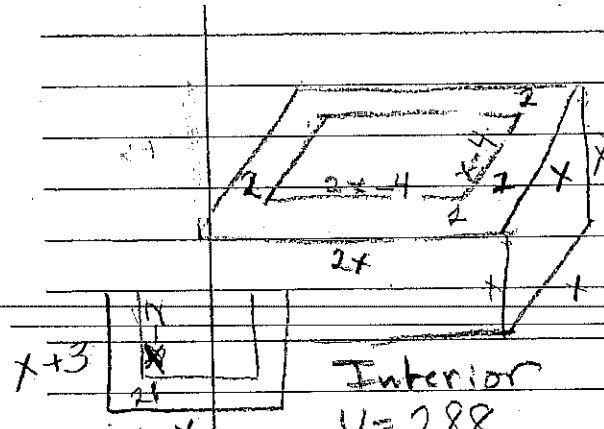
$-(8x + 28)$

-3

$$(2x^4 + x^3 - 31x^2 - 27x + 25) = (2x+7)(x^3 - 3x^2 - 5x + 4) - 3$$

6.4 #10

10. A new rectangular holding tank is being built. The tank's sides and bottom should be 2 feet thick. Its outer length should be twice its outer width. The outer height is the same measure as the outer width. What should the outer dimensions of the tank be if it is to have a volume of 288 cubic feet?



Outer dem.

$h = x$

$w = x$

$l = 2x$

Interior

$V = 288$

$h = (x-2)$

$w = x-4$

$l = 2x-4$

$288 = (2x-4)(x-4)(x-2)$

$288 = (2x-4)(x-4)(x-2)$

$288 = (2x-4)(x^2-6x+8)$

$288 = 2x^3 - 12x^2 + 16x - 4x^2 + 24x - 32$

$0 = (2x^3 - 16x^2) + 40x - 320$

$0 = 2x^2(x-8) + 40(x-8)$

$0 = (2x^2 + 40)(x-8)$

$x = 8$ only real

height = 8

length = 16

width = 8